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M-branes, anti-M-branes and nonextremal black holes

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Abstract

An M-brane and anti-M-brane scheme is proposed to study nonextremal $4D$ and $5D$ black holes. The improved nonextremal intersecting M-brane solutions proposed here, involve two sets of harmonic functions. The constraints among the pressures are found, and new features in the M-brane and anti-M-brane picture are demonstrated, which resolve the discrepancy in the number of free parameters in the D-brane picture. In terms of the "numbers" of M-branes and anti-M-branes, the prefactors of the entropies are found to be model independent, and the Bekenstein-Hawking entropy assumes the duality invariant form which is consistent with the microscopic explanation of the black hole entropy.

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1 Introduction

Recently, more and more evidence has accumulated to show that the best candidate for a unified theory underlying all physical phenomena is no longer $D = 10$ superstring theory but rather $D = 11$ M-theory which generalizes known string theories [1], and $D = 11$ supergravity can be regarded as a low-energy effective field theory of the fundamental M-theory. The precise formulation of M-theory is not clear, but 2-branes and 5-branes play an important role in the $D = 11$ supergravity theory owing to the 4-form field strength F_4 [2]. The supersymmetric BPS-saturated p-brane solutions of low-dimensional theories can be understood as reductions of the basic $D = 11$ M-branes, i.e. as 2-branes [3] and 5-branes [4] and their combinations [5]. The generalization to a number of different harmonic functions specifying intersecting BPS-saturated M-branes and a construction of new intersecting p-brane solutions in $D \leq 11$ were given in [6]. It has been shown that there exists a simple harmonic function rule which governs the construction of composite supersymmetric p-brane solutions in both $D = 10$ and $D = 11$, and a separate harmonic function is assigned to each constituent $\frac{1}{2}$ supersymmetric p-brane [6]. Later, the extremal supersymmetric BPS-saturated intersecting M-brane solution was generalized to the nonextremal case [7]. The resulting nonextremal intersecting M-brane solutions are no longer supersymmetric, but they can be constructed as a deformation of extremal solutions, parametrised in terms of several one-center harmonic functions and the Schwarzschild solution, parametrised in terms of m. The nonextremal configurations of intersecting M-branes can be related to nonextremal black holes in dimensions $D = 4$, $D = 5$ and $6 \leq D \leq 9$, by dimensional reduction along internal M-brane directions. In [7], the nonextremal black holes were explained as nonextremal intersecting M-branes which has nothing to do with the D-brane-antibrane picture [8, 9, 10]. On the other hand, the statistical origin of the Bekenstein-Hawking entropy of certain extremal black holes in string theory can be elucidated by the D-brane technique [11]-[18], and in particular, the nonextremal black holes can be decomposed into a collection of D-branes and anti-D-branes [8, 9, 10]. Then one may ask whether it is possible to discuss the nonextremal black holes in M-theory from the M-brane and anti-M-brane approach by trading the parameters of the general solution for the "numbers" of M-branes and anti-M-branes. As shown below, this can be done only by relating the unconstrained pressures, ADM mass and gauge charges of the black hole to the "numbers" of a collection of noninteracting constituent branes and antibranes. In the D-brane picture, the number of free parameters should consist of the "numbers" of D-branes and anti-D-branes plus the values of moduli [19], but it turns out that for the black hole solutions in string theories, when the "numbers" of D-branes and anti-D-branes are given, the values of moduli will be fixed, that is, they are determined by the actual number of branes of each type [8]. As a result, a dicrepancy in the number of free parameters appears [8]. In fact, seen from the black hole picture, the values of moduli are neither completely arbitrary, nor fixed, since if they are too large, the solutions become classically unstable [20]. However, how the above contradiction can be reconciled in the framework of D-branes is unclear.

In the present paper, the D-brane and anti-D-brane picture is extended to M-theory, i.e., M-branes and anti-M-branes. The nonextremal $4D$ and $5D$ black holes obtained upon toroidal compactification from the nonextremal intersecting M-brane solutions are first identified as the composition of M-branes and anti-M-branes. In a unified frame of $D = 11$ M-theory, the "numbers" of M-branes and anti-M-branes are defined by matching thermodynamic properties of the black hole to thermodynamic properties of a collection of noninteracting M-branes and anti-M-branes. In our improved nonextremal intersecting M-brane solutions, two sets of harmonic functions $\sigma_i(\omega_i)$ and $\tilde{\sigma}_i(\tilde{\omega}_i)$ are first introduced, and the field thrength F_4 for 2-branes is different from that in [7] where only one set of harmonic functions T_i^{-1} was given and the other set of complicated function $T_i^{'-1}$ were not harmonic ones. From our construction for the field strength F_4, it is easy to calculate explicitly the electric charges, in fact the electric and magnetic charges can be evaluated in a systematic way in eleven dimensions. It is first found that there exist constraints among the pressures, and only the unconstrained pressures, ADM mass and gauge charges match the "numbers" of M-branes and anti-M-branes. Unlike the D-brane and anti-D-brane picture where the moduli are only functions of the "numbers" of D-branes and anti-D-branes [8]-[10], there are new features in the M-brane and anti-M-brane picture: 1) as shown in section 2.1, there are constraints among the moduli, and some moduli can be chosen as free parameters; 2) the moduli depend not only on the "numbers" of M-branes and anti-M-branes, but also on the nonextremality parameter m. The reason for this is that the masses of M-branes are not independent parameters, instead they are proportional to the common mass parameter m, and the configurations of the nonextremal intersecting M-brane solutions should be viewed as "bound-state" configurations. The constraints among the pressures and moduli found here are different from those in [10], where the appearance of their constraints are due to introducing the auxiliary SU(8) multiplet via E(7) symmetry. In the present M-brane picture, when the "numbers" of M-branes and anti-M-branes are kept, the values of moduli can not be fixed completely, which resolves the discrepancy in the number of free parameters in the D-brane picture. As we know, when the black hole entropies are expressed in terms the original parameters of the solutions, the prefactors of the entropies depend on the dimensions of the black hole [7, 21]. In terms of the "numbers" of M-branes and anti-M-branes, the Bekenstein-Hawking entropy takes the duality invariant form which is consistent with the microscopic explanation of the black hole entropy, and the prefactors of the entropies are found to be 2π, which is independent of the dimensions of the black holes. In [8, 9], only one model was discussed in each paper, and the authors fit their conventions in different ways so that the prefactors are 2π. However, in [21], since no unified frame was exploited, the prefactors are $2\pi, 4\pi$ respectively in $D = 4, 5$ dimensions. Here we study different models in a *unified* frame of $D = 11$ M-theory; thus the universality of the prefactors is nontrivial. The same prefactors of the entropies between two black holes in the same dimensions obtained from two different nonextremal intersecting M-brane configurations imply that these two resulting black hole backgrounds are related by the symmetry transformation of the M-theory, which is a combination of T-duality and SL(2,Z) symmetry of the $D = 10$

type IIB theory lifted to $D=11$ M-theory. Even though the general formula for the statistical entropy cannot be derived from a counting of states, this can be done in certain limits corresponding to near-extremal black holes. By exploiting U-duality, one can interchange the different branes, then the resulting equation for microscopic entropy is duality invariant, which agrees with different nonextremal limits.

The layout of the paper is as follows. In the next section we consider two $4D$ nonextremal black holes obtained upon toroidal compactification from the nonextremal intersecting M-brane configurations $2\perp 2\perp 5\perp 5$ and "boosted" $5\perp 5\perp 5$, and define the "numbers" of M-branes and anti-M-branes in a unified frame of $D=11$ M-theory. We display the constraints among the pressures and moduli, and express the Bekenstein-Hawking entropies in terms of the "numbers" of M-branes and anti-M-branes. In section 3, we discuss two nonextremal $5D$ black holes obtained from the configurations $2\perp 2\perp 2$ and "boosted" $2\perp 5$ in a way similar to that of section 2. In section 4, we explain the microscopic origin of the Bekenstein-Hawking entropy for the nonextremal black holes. Finally, in section 5, we present our conclusions.

2 $D=4$ nonextremal black holes

We consider nonextremal $4D$ black holes which can be obtained upon toroidal compactification from two kinds of nonextremal intersecting M-brane configurations, i.e. $2\perp 2\perp 5\perp 5$ and the "boosted" $5\perp 5\perp 5$ [7, 22].

2.1 The $2\perp 2\perp 5\perp 5$, $D=11$, solution

From the algorithm which leads to the nonextremal version of a given extremal intersecting M-brane solution [7], the $2\perp 2\perp 5\perp 5$ configuration in $D=11$ M-theory is described by

$$\begin{aligned} ds^2_{11} = (\sigma_1\sigma_2)^{\frac{1}{3}}(\omega_1\omega_2)^{\frac{2}{3}}\Big[&-(\sigma_1\sigma_2\omega_1\omega_2)^{-1}\left(1-\frac{2m}{R}\right)dt^2 \\ &+(\sigma_1\omega_1)^{-1}dz_1^2+(\sigma_1\omega_2)^{-1}dz_2^2+(\sigma_2\omega_1)^{-1}dz_3^2+(\sigma_2\omega_2)^{-1}dz_4^2 \\ &+(\omega_1\omega_2)^{-1}(dz_5^2+dz_6^2+dz_7^2)+\left(1-\frac{2m}{R}\right)^{-1}dR^2+R^2d\Omega_2^2\Big] \end{aligned} \tag{1}$$

The associated gauge field which must be a 4-form is given by

$$F_4 = F_4(\sigma_1)+F_4(\sigma_2)+F_4(\omega_1)+F_4(\omega_2)$$

with

$$\begin{aligned} F_4(\sigma_1) &= 3dt\wedge(\sigma_1^{-2}d\tilde{\sigma}_1)\wedge dz_1\wedge dz_2 \\ F_4(\sigma_2) &= 3dt\wedge(\sigma_2^{-2}d\tilde{\sigma}_2)\wedge dz_3\wedge dz_4 \\ F_4(\omega_1) &= 3(*d\tilde{\omega}_1)\wedge dz_2\wedge dz_4 \\ F_4(\omega_2) &= 3(*d\tilde{\omega}_2)\wedge dz_1\wedge dz_3 \end{aligned} \tag{2}$$

where the dual form $*$ is defined in the asymptotically flat 3-dimensional transverse space, and $\sigma_i(\tilde{\sigma}_i), \omega_i(\tilde{\omega}_i)$ are harmonic functions corresponding to the 2-branes, 5-branes respectively,

$$\begin{aligned} \sigma_i &= 1 + \frac{2m\sinh^2\alpha_i}{R}, \quad \tilde{\sigma}_i = 1 + \frac{m\sinh 2\alpha_i}{R} \\ \omega_i &= 1 + \frac{2m\sinh^2\beta_i}{R}, \quad \tilde{\omega}_i = 1 + \frac{m\sinh 2\beta_i}{R}, \quad i = 1, 2 \end{aligned} \tag{3}$$

where α_i and β_i are related to the electric and magnetic charges and m is the nonextremality parameter. Here we note that our improved nonextremal intersecting M-brane solution for the field strength $F_4(\sigma_i)$ is different from that in [7].

To obtain the electric charges, we calculate the quantities $\hat{*}F_4(\sigma_1), \hat{*}F_4(\sigma_2)$, where $\hat{*}$ denotes the $D = 11$ Hodge dual. After a series of calculational steps, one has

$$\begin{aligned} \hat{*}F_4(\sigma_1) &= m(\sinh 2\alpha_1)\epsilon_2 \wedge dz_3 \wedge dz_4 \wedge dz_5 \wedge dz_6 \wedge dz_7 \\ \hat{*}F_4(\sigma_2) &= m(\sinh 2\alpha_2)\epsilon_2 \wedge dz_1 \wedge dz_2 \wedge dz_5 \wedge dz_6 \wedge dz_7 \end{aligned} \tag{4}$$

where ϵ_2 is the area 2-form of S^2. Then the electric and magnetic charges can be defined as

$$\begin{aligned} q_e^{(1)} &= \frac{1}{\sqrt{2}\kappa}\int \hat{*}F_4(\sigma_1) = \frac{4\pi m\sinh 2\alpha_1}{\sqrt{2}\kappa} l_3 l_4 V_3 \\ q_e^{(2)} &= \frac{1}{\sqrt{2}\kappa}\int \hat{*}F_4(\sigma_2) = \frac{4\pi m\sinh 2\alpha_2}{\sqrt{2}\kappa} l_1 l_2 V_3 \\ q_m^{(1)} &= \frac{1}{\sqrt{2}\kappa}\int F_4(\omega_1) = \frac{4\pi m\sinh 2\beta_1}{\sqrt{2}\kappa} l_2 l_4 \\ q_m^{(2)} &= \frac{1}{\sqrt{2}\kappa}\int F_4(\omega_2) = \frac{4\pi m\sinh 2\beta_2}{\sqrt{2}\kappa} l_1 l_3 \end{aligned} \tag{5}$$

where $\kappa^2/8\pi$ is Newton's constant in 11 dimensions, and in the above calculation, we have assumed the internal coordinates $z_i, i = 1, ..., 4$, are periodically identified with period l_i, and $z_i, i = 5, 6, 7$, are identified with period $(V_3)^{\frac{1}{3}}$. As a result, the solution (1) has ten parameters: $m, \alpha_i, \beta_i, i = 1, 2, l_j, j = 1, ..., 4$ and V_3.

The ADM mass of the solution (1) is [7, 23]

$$M_{ADM} = \frac{2\pi m}{\kappa^2} l_1 l_2 l_3 l_4 V_3 (\cosh 2\alpha_1 + \cosh 2\alpha_2 + \cosh 2\beta_1 + \cosh 2\beta_2) \tag{6}$$

In eleven dimensions, besides the ADM mass and the gauge charges, the black hole is also characterized by pressures which are related to the asymptotic fall-off of the coefficents of $(dz_i)^2, i = 1, ..., 5$, in the solution (1). These pressures P_i are given by [8]

$$P_1 = \frac{2\pi m}{\kappa^2} l_1 l_2 l_3 l_4 V_3 (-2\cosh 2\alpha_1 + \cosh 2\alpha_2 - \cosh 2\beta_1 + 2\cosh 2\beta_2)$$

$$
\begin{aligned}
P_2 &= \frac{2\pi m}{\kappa^2} l_1 l_2 l_3 l_4 V_3(-2\cosh 2\alpha_1 + \cosh 2\alpha_2 + 2\cosh 2\beta_1 - \cosh 2\beta_2) \\
P_3 &= \frac{2\pi m}{\kappa^2} l_1 l_2 l_3 l_4 V_3(\cosh 2\alpha_1 - 2\cosh 2\alpha_2 - \cosh 2\beta_1 + 2\cosh 2\beta_2) \\
P_4 &= \frac{2\pi m}{\kappa^2} l_1 l_2 l_3 l_4 V_3(\cosh 2\alpha_1 - 2\cosh 2\alpha_2 + 2\cosh 2\beta_1 - \cosh 2\beta_2) \\
P_5 &= \frac{2\pi m}{\kappa^2} l_1 l_2 l_3 l_4 V_3(\cosh 2\alpha_1 + \cosh 2\alpha_2 - \cosh 2\beta_1 - \cosh 2\beta_2)
\end{aligned} \tag{7}
$$

Eqs. (5-7) suggest that the gauge charges, ADM mass and the pressures $P_i, i = 1, ..., 5$, can replace the ten parameters in the solution (1): $m, \alpha_i, \beta_i, i = 1, 2, l_j, j = 1, ..., 4$, and V_3. However, the pressures P_i are restricted by the following two constraints:

$$
P_1 + P_4 + P_5 = 0, \quad P_2 + P_3 + P_5 = 0 \tag{8}
$$

which indicate the exchange symmetry with respect to the two 2-branes ($\sigma_1 \rightleftharpoons \sigma_2$), and the two 5-branes ($\omega_1 \rightleftharpoons \omega_2$), so that the number of independent pressures is 5-2=3. In fact, thanks to these two constraints, three unconstrained pressures, the ADM mass and the four gauge charges can match the eight "numbers" of two 2-branes (anti-2-branes) and two 5-branes (anti-5-branes). To see how this matching works, we calculate the values of the ADM mass and pressures for each type of brane in the four extremal limits: $m \to 0, \alpha_i(\beta_i) \to \pm\infty$ with $q_e^{(i)}(q_m^{(i)})$ and $\alpha_j(\beta_j)(j \neq i)$ fixed. For one first 2-brane ($\alpha_1 \to \pm\infty$), the ADM mass and pressures reduce to

$$
M_{ADM} = -\frac{1}{2}P_1 = -\frac{1}{2}P_2 = P_3 = P_4 = P_5 = \frac{l_1 l_2 \hat{e}}{\sqrt{2}\kappa} \tag{9}
$$

while for the second 2-brane ($\alpha_2 \to \pm\infty$), they become

$$
M_{ADM} = P_1 = P_2 = -\frac{1}{2}P_3 = -\frac{1}{2}P_4 = P_5 = \frac{l_3 l_4 \hat{e}}{\sqrt{2}\kappa} \tag{10}
$$

For a single first 5-brane ($\beta_1 \to \pm\infty$), one obtains

$$
M_{ADM} = -P_1 = \frac{1}{2}P_2 = -P_3 = \frac{1}{2}P_4 = -P_5 = \frac{l_1 l_3 V_3}{\sqrt{2}\kappa}\hat{e}_m \tag{11}
$$

and for the single second 5-brane ($\beta_2 \to \pm\infty$) they can be written as

$$
M_{ADM} = \frac{1}{2}P_1 = -P_2 = \frac{1}{2}P_3 = -P_4 = -P_5 = \frac{l_2 l_4 V_3}{\sqrt{2}\kappa}\hat{e}_m \tag{12}
$$

where $\hat{e}, \hat{e}_m$ are the unit charges of the 2-brane and the 5-brane [24]

$$
\begin{aligned}
\hat{e} &= \sqrt{2}(2\kappa\pi^2)^{1/3}, \\
\hat{e}_m &= \sqrt{2}(\pi/2\kappa)^{1/3}
\end{aligned} \tag{13}
$$

which satisfy the Dirac condition:

$$\hat{e} \cdot \hat{e}_m = 2\pi \tag{14}$$

Comparing Eqs. (5-8) with (9-12), we can trade eight parameters of the original ten parameters in the solution (1) for the eight "numbers": $N_2^{(1)}, \bar{N}_2^{(1)}, N_2^{(2)}, \bar{N}_2^{(2)}, N_5^{(1)}, \bar{N}_5^{(1)}, N_5^{(2)}, \bar{N}_5^{(2)}$, which are the "numbers" of 2-branes, anti-2-branes, 5-branes, and anti-5-branes. In particular, this can be done by matching three unconstrained pressures, the ADM mass and four gauge charges to those of a collection of noninteracting branes. The definitions of the N's are

$$\begin{aligned}
N_2^{(1)} &= \frac{\sqrt{2}\pi m l_3 l_4 V_3}{\kappa \hat{e}} e^{2\alpha_1}, \quad \bar{N}_2^{(1)} = \frac{\sqrt{2}\pi m l_3 l_4 V_3}{\kappa \hat{e}} e^{-2\alpha_1}, \\
N_2^{(2)} &= \frac{\sqrt{2}\pi m l_1 l_2 V_3}{\kappa \hat{e}} e^{2\alpha_2}, \quad \bar{N}_2^{(2)} = \frac{\sqrt{2}\pi m l_1 l_2 V_3}{\kappa \hat{e}} e^{-2\alpha_2}, \\
N_5^{(1)} &= \frac{\sqrt{2}\pi m l_2 l_4}{\kappa \hat{e}_m} e^{2\beta_1}, \quad \bar{N}_5^{(1)} = \frac{\sqrt{2}\pi m l_2 l_4}{\kappa \hat{e}_m} e^{-2\beta_1}, \\
N_5^{(2)} &= \frac{\sqrt{2}\pi m l_1 l_3}{\kappa \hat{e}_m} e^{2\beta_2}, \quad \bar{N}_5^{(2)} = \frac{\sqrt{2}\pi m l_1 l_3}{\kappa \hat{e}_m} e^{-2\beta_2}
\end{aligned} \tag{15}$$

In terms of these "numbers" of M-branes and anti-M-branes, the gauge charges are

$$\begin{aligned}
q_e^{(i)} &= (N_2^{(i)} - \bar{N}_2^{(i)})\hat{e} \\
q_m^{(i)} &= (N_5^{(i)} - \bar{N}_5^{(i)})\hat{e}_m, \quad i = 1, 2,
\end{aligned} \tag{16}$$

the ADM mass is

$$\begin{aligned}
M_{ADM} = \frac{l_1 l_2 \hat{e}}{\sqrt{2}\kappa}\left(N_2^{(1)} + \bar{N}_2^{(1)}\right) + \frac{l_3 l_4 \hat{e}}{\sqrt{2}\kappa}\left(N_2^{(2)} + \bar{N}_2^{(2)}\right) \\
+\frac{l_1 l_3 V_3 \hat{e}_m}{\sqrt{2}\kappa}\left(N_5^{(1)} + \bar{N}_5^{(1)}\right) + \frac{l_2 l_4 V_3 \hat{e}_m}{\sqrt{2}\kappa}\left(N_5^{(2)} + \bar{N}_5^{(2)}\right),
\end{aligned} \tag{17}$$

and the other parameters are

$$V_3 = \frac{\hat{e}}{e_m}\left(\frac{N_2^{(1)}\bar{N}_2^{(1)}N_2^{(2)}\bar{N}_2^{(2)}}{N_5^{(1)}\bar{N}_5^{(1)}N_5^{(2)}\bar{N}_5^{(2)}}\right)^{\frac{1}{4}} \tag{18}$$

$$l_3 = \left(\frac{N_2^{(1)}\bar{N}_2^{(1)}N_5^{(2)}\bar{N}_5^{(2)}}{N_2^{(2)}\bar{N}_2^{(2)}N_5^{(1)}\bar{N}_5^{(1)}}\right)^{\frac{1}{4}} l_2 \tag{19}$$

$$l_4 = \left(\frac{N_2^{(1)}\bar{N}_2^{(1)}N_5^{(1)}\bar{N}_5^{(1)}}{N_2^{(2)}\bar{N}_2^{(2)}N_5^{(2)}\bar{N}_5^{(2)}}\right)^{\frac{1}{4}} l_1, \tag{20}$$

$$m=\frac{\kappa\hat{e}_m}{\sqrt{2}\pi}\left(\frac{N_2^{(2)}\bar{N}_2^{(2)}N_5^{(1)}\bar{N}_5^{(1)}N_5^{(2)}\bar{N}_5^{(2)}}{N_2^{(1)}\bar{N}_2^{(1)}}\right)^{\frac{1}{4}}(l_1l_2)^{-1} \tag{21}$$

Eqs.(15-21) show that the original ten parameters in the solution (1) can be replaced by eight "numbers" of two 2-branes and anti-2-branes, two 5-branes and anti-5-branes, plus the moduli l_1, l_2. In particular, from Eqs. (19, 20) we find that there are two constraints among the values of moduli $l_i, i = 1, ..., 4$. Different from the p-brane solutions in $D = 10$ string theories [8, 9], here even the number of branes of each type is given and the values of moduli l_i are not completely fixed, which is consistent with the black hole picture and resolves the discrepancy in the number of free parameters in the D-brane picture. Since the four M-branes are orthogonal and the mass of each M-brane is proportional to the common mass parameter m, such nonextremal intersecting M-brane configurations can be viewed as "bound-state" configurations.

Upon toroidal compactification to four dimensions, the solution (1) is reduced to the following Einstein-frame metric [7]

$$ds_4^2=-f(R)\left(1-\frac{2m}{R}\right)dt^2+f^{-1}(R)\left[\left(1-\frac{2m}{R}\right)^{-1}dR^2+R^2d\Omega_2^2\right] \tag{22}$$

with

$$f(R)=\frac{R^2}{\left[(R+2m\sinh^2\alpha_1)(R+2m\sinh^2\alpha_2)(R+2m\sinh^2\beta_1)(R+2m\sinh^2\beta_2)\right]^{\frac{1}{2}}} \tag{23}$$

From (1), (22) and (23), we get the Bekenstein-Hawking entropy

$$S_{BH}=\frac{2\pi A_9}{\kappa^2}=\frac{2\pi A_2}{\kappa_4^2}=\frac{32\pi^2m^2}{\kappa^2}l_1l_2l_3l_4V_3\cosh\alpha_1\cosh\alpha_2\cosh\beta_1\cosh\beta_2 \tag{24}$$

where A is the area of the horizon ($R = 0$), and $\kappa_4^2/8\pi$ is Newton's constant in $D = 4$ dimensions. The Hawking temperature can be read off from Eqs. (22, 23)

$$T_H=(8\pi m\cosh\alpha_1\cosh\alpha_2\cosh\beta_1\cosh\beta_2)^{-1} \tag{25}$$

In terms of N's, the black hole entropy (24) is reexpressed as

$$S_{BH}=2\pi\left(\sqrt{N_2^{(1)}}+\sqrt{\bar{N}_2^{(1)}}\right)\left(\sqrt{N_2^{(2)}}+\sqrt{\bar{N}_2^{(2)}}\right)\left(\sqrt{N_5^{(1)}}+\sqrt{\bar{N}_5^{(1)}}\right)\left(\sqrt{N_5^{(2)}}+\sqrt{\bar{N}_5^{(2)}}\right) \tag{26}$$

which shows that there are ten parameters in the solution (1) which can be replaced by $N_2^{(i)}, \bar{N}_2^{(i)}, N_5^{(i)}, \bar{N}_5^{(i)}, i = 1, 2$ plus l_1, l_2, the black hole entropy being independent of the moduli l_1, l_2, and this topological character of the entropy is consistent with that emphasized in [25, 26].

2.2 The "boosted" $5 \perp 5 \perp 5$ configuration

The second nonextremal intersecting M-brane solution in $D = 11$ dimensions corresponds to three 5-branes, each pair intersecting at a 3-brane, with a boost along a string common to three 3-branes. The nonextremal background for the $5 \perp 5 \perp 5$ configuration with a boost is given by

$$
\begin{aligned}
ds_{11}^2 &= (\omega_1\omega_2\omega_3)^{\frac{2}{3}}\Big\{(\omega_1\omega_2\omega_3)^{-1}\left[(-\eta^{-1}\left(1-\frac{2m}{R}\right)dt^2+\eta(dz_1')^2\right] \\
&\quad +(\omega_2\omega_3)^{-1}(dz_2^2+dz_3^2)+(\omega_3\omega_1)^{-1}(dz_4^2+dz_5^2)+(\omega_1\omega_2)^{-1}(dz_6^2+dz_7^2) \\
&\quad +\left(1-\frac{2m}{R}\right)^{-1}dR^2+R^2d\Omega_2^2\Big\}, \\
F_4 &= F_4(\omega_1)+F_4(\omega_2)+F_4(\omega_3)
\end{aligned}
\tag{27}
$$

with

$$
\begin{aligned}
F_4(\omega_1) &= 3(*d\tilde{\omega}_1)\wedge dz_2\wedge dz_3, \\
F_4(\omega_2) &= 3(*d\tilde{\omega}_2)\wedge dz_4\wedge dz_5, \\
F_4(\omega_3) &= 3(*d\tilde{\omega}_3)\wedge dz_6\wedge dz_7,
\end{aligned}
\tag{28}
$$

where the harmonic functions $\eta, \omega_i(\tilde{\omega}_i)$ and the differential dz_1' are defined as

$$
\begin{aligned}
\eta &= 1+\frac{2m\sinh^2\alpha}{R}, \\
\omega_i &= 1+\frac{2m\sinh^2\beta_i}{R}, \quad \tilde{\omega}_i = 1+\frac{m\sinh 2\beta_i}{R}, \quad i=1,2,3 \\
dz_1' &= dz_1-\frac{m\sinh 2\alpha}{R}\eta dt
\end{aligned}
\tag{29}
$$

The electric and three magnetic charges are [27]

$$
\begin{aligned}
Q &= m\sinh 2\alpha = \frac{2\kappa_4^2}{\omega_2}\cdot\frac{2\pi(n-\bar{n})}{l_1} = \frac{\kappa^2(n-\bar{n})}{l_1^2 S_{23}S_{45}S_{67}} \\
q_m^{(1)} &= \frac{1}{\sqrt{2}\kappa}\int F_4(\omega_1) = \frac{4\pi m\sinh 2\beta_1}{\sqrt{2}\kappa}S_{23} \\
q_m^{(2)} &= \frac{1}{\sqrt{2}\kappa}\int F_4(\omega_2) = \frac{4\pi m\sinh 2\beta_2}{\sqrt{2}\kappa}S_{45} \\
q_m^{(3)} &= \frac{1}{\sqrt{2}\kappa}\int F_4(\omega_3) = \frac{4\pi m\sinh 2\beta_3}{\sqrt{2}\kappa}S_{67}
\end{aligned}
\tag{30}
$$

where $n, \bar{n}$ are positive integers and the brane-coordinates $z_1, z_{2,3}, z_{4,5}, z_{6,7}$ are compactified on $S^1\times T^2\times T^2\times T^2$, the circumference of circle and areas of three 2-tori being $l_1, S_{23}, S_{45}, S_{67}$ respectively, and the solution (27) has nine parameters: $m, \alpha, \beta_i, i=1,2,3, l_1, S_{23}, S_{45}, S_{67}$.

The ADM mass and the pressures of the solution (27) are found to be

$$\begin{aligned}
M_{ADM} &= \frac{2\pi m}{\kappa^2} l_1 S_{23} S_{45} S_{67} (\cosh 2\alpha + \cosh 2\beta_1 + \cosh 2\beta_2 + \cosh 2\beta_3), \\
P_1 &= \frac{2\pi m}{\kappa^2} l_1 S_{23} S_{45} S_{67} (3\cosh 2\alpha - \cosh 2\beta_1 - \cosh 2\beta_2 - \cosh 2\beta_3), \\
P_{23} &= \frac{2\pi m}{\kappa^2} l_1 S_{23} S_{45} S_{67} (2\cosh 2\beta_1 - \cosh 2\beta_2 - \cosh 2\beta_3), \\
P_{45} &= \frac{2\pi m}{\kappa^2} l_1 S_{23} S_{45} S_{67} (-\cosh 2\beta_1 + 2\cosh 2\beta_2 - \cosh 2\beta_3), \\
P_{67} &= \frac{2\pi m}{\kappa^2} l_1 S_{23} S_{45} S_{67} (-\cosh 2\beta_1 - \cosh 2\beta_2 + 2\cosh 2\beta_3),
\end{aligned} \tag{31}$$

Eqs. (30, 31) suggest that the nine parameters in the solution (27) can be replaced by other nine quantities: $Q, q_m^{(i)}, i = 1, 2, 3, P_1, P_{23}, P_{45}, P_{67}$. But the pressures are not independent and are related through the constraint

$$P_{23} + P_{45} + P_{67} = 0 \tag{32}$$

which reflects the exchange symmetry of the solution (27) with respect to three 5-branes, so that the number of unconstrained pressures is $4 - 1 = 3$. Following a discussion similar to that of section 2.1, the "numbers" of three 5-branes, anti-5-branes, right-moving strings and left-moving strings are defined by

$$\begin{aligned}
N_5^{(1)} &= \frac{\sqrt{2}\pi m S_{23}}{\kappa \hat{e}_m} e^{2\beta_1}, \quad \bar{N}_5^{(1)} = \frac{\sqrt{2}\pi m S_{23}}{\kappa \hat{e}_m} e^{-2\beta_1}, \\
N_5^{(2)} &= \frac{\sqrt{2}\pi m S_{45}}{\kappa \hat{e}_m} e^{2\beta_2}, \quad \bar{N}_5^{(2)} = \frac{\sqrt{2}\pi m S_{45}}{\kappa \hat{e}_m} e^{-2\beta_2}, \\
N_5^{(3)} &= \frac{\sqrt{2}\pi m S_{67}}{\kappa \hat{e}_m} e^{2\beta_3}, \quad \bar{N}_5^{(3)} = \frac{\sqrt{2}\pi m S_{67}}{\kappa \hat{e}_m} e^{-2\beta_3}, \\
n &= \frac{m l_1^2 S_{23} S_{45} S_{67}}{2\kappa^2} e^{2\alpha}, \quad \bar{n} = \frac{m l_1^2 S_{23} S_{45} S_{67}}{2\kappa^2} e^{-2\alpha}
\end{aligned} \tag{33}$$

The gauge charges are

$$\begin{aligned}
q_m^{(i)} &= \left(N_5^{(i)} - \bar{N}_5^{(i)}\right) \hat{e}_m, \quad i = 1, 2, 3 \\
q_e &= (n - \bar{n})\hat{e},
\end{aligned} \tag{34}$$

and the ADM mass is

$$\begin{aligned}
M_{ADM} &= \frac{\hat{e}_m}{\sqrt{2}\kappa} \Big[l_1 S_{45} S_{67} \left(N_5^{(1)} + \bar{N}_5^{(1)}\right) + l_1 S_{23} S_{67} \left(N_5^{(2)} + \bar{N}_5^{(2)}\right) \\
&\quad l_1 S_{23} S_{45} \left(N_5^{(3)} + \bar{N}_5^{(3)}\right) + \frac{2\sqrt{2}\pi\kappa}{\hat{e}_m l_1}(n + \bar{n}) \Big],
\end{aligned} \tag{35}$$

and the values of the moduli are

$$
\begin{aligned}
l_1^2 &= \frac{\sqrt{2}\pi m^2}{\kappa \hat{e}_m^3}\left(\frac{n\bar{n}}{\prod_{i=1}^{3} N_5^{(i)}\bar{N}_5^{(i)}}\right)^{\frac{1}{2}}, \\
S_{23} &= \frac{\kappa \hat{e}_m}{\sqrt{2}\pi}\sqrt{N_5^{(1)}\bar{N}_5^{(1)}}\, m^{-1}, \\
S_{45} &= \frac{\kappa \hat{e}_m}{\sqrt{2}\pi}\sqrt{N_5^{(2)}\bar{N}_5^{(2)}}\, m^{-1}, \\
S_{67} &= \frac{\kappa \hat{e}_m}{\sqrt{2}\pi}\sqrt{N_5^{(3)}\bar{N}_5^{(3)}}\, m^{-1},
\end{aligned} \tag{36}
$$

which show that the moduli depend not only on the "numbers" of M-branes and anti-M-branes, but also on the nonextremality parameter m. Therefore, when the "numbers" of M-branes and anti-M-branes are kept fixed, the values of the moduli cannot be determined completely, which reconciles the apparent contradiction between the number of free parameters of the D-brane picture and those of the black hole one. Furthermore, from (33-36) we find that the eight "numbers" of the three 5-branes, anti-5-branes, right-moving strings and left-moving strings, plus the nonextremality parameter m can replace the original nine parameters in solution (27).

When compactified to four dimensions, the solution (27) is reduced to (22) with

$$
f(R) = R^2\left[(R+2m\sinh^2\alpha)(R+2m\sinh^2\beta_1)(R+2m\sinh^2\beta_2)(R+2m\sinh^2\beta_3)\right]^{-\frac{1}{2}} \tag{37}
$$

Then the Bekenstein-Hawking entropy can derived from (22) and (37) to be

$$
S_{BH} = \frac{2\pi A_9}{\kappa^2} = \frac{2\pi A_2}{\kappa_4^2} = \frac{32\pi^2 m^2}{\kappa^2} l_1 S_{23} S_{45} S_{67} \cosh\alpha\cosh\beta_1\cosh\beta_2\cosh\beta_3 \tag{38}
$$

and the Hawking temperature is

$$
T_H = (8\pi m\cosh\alpha\cosh\beta_1\cosh\beta_2\cosh\beta_3)^{-1} \tag{39}
$$

In terms of N's, Eq. (38) can be rewritten as

$$
S_{BH} = 2\pi\left(\sqrt{N_5^{(1)}}+\sqrt{\bar{N}_5^{(1)}}\right)\left(\sqrt{N_5^{(2)}}+\sqrt{\bar{N}_5^{(2)}}\right)\left(\sqrt{N_5^{(3)}}+\sqrt{\bar{N}_5^{(3)}}\right)\left(\sqrt{n}+\sqrt{\bar{n}}\right) \tag{40}
$$

which shows that even here there are nine free parameters: $n, \bar{n}, N_5^{(i)}, \bar{N}_5^{(i)}, i = 1,2,3$, plus m, the Bekenstein-Hawking entropy being independent of the parameter m. The same prefactors in (26) and (40) imply that the above two nonextremal $4D$ black hole backgrounds obtained from $2 \perp 2 \perp 5 \perp 5$ and "boosted" $5 \perp 5 \perp 5$ nonextremal intersecting M-brane configurations can be related by the symmetry transformation of M-theory, which is a combination of T-duality and SL(2,Z) symmetry of $D = 10$ type IIB theory lifted to $D = 11$ M-theory.

3 Nonextremal 5D black holes

The nonextremal $5D$ black holes with three independent charges can be obtained from two different nonextremal intersecting M-brane solutions: $2 \perp 2 \perp 2$ (three 2-branes intersecting at a point), and "boosted" $2 \perp 5$ (intersecting 2-brane and 5-brane with a momentum along the common string).

3.1 The $2 \perp 2 \perp 2$, $D = 11$ solution

The nonextremal intersecting M-brane solution corresponding to the $2 \perp 2 \perp 2$ configuration is given by

$$\begin{aligned} ds_{11}^2 &= (\sigma_1\sigma_2\sigma_3)^{\frac{1}{3}}\Big[-(\sigma_1\sigma_2\sigma_3)^{-1}\left(1-\frac{2m}{R^2}\right)dt^2+\sigma_1^{-1}(dz_1^2+dz_2^2) \\ &\quad +\sigma_2^{-1}(dz_3^2+dz_4^2)+\sigma_3^{-1}(dz_5^2+dz_6^2)+\left(1-\frac{2m}{R^2}\right)^{-1}dR^2+R^2d\Omega_3^2\Big] \end{aligned}$$

and

$$F_4 = F_4(\sigma_1)+F_4(\sigma_2)+F_4(\sigma_3) \tag{41}$$

with

$$\begin{aligned} F_4(\sigma_1) &= 3dt\wedge(\sigma_1^{-2}d\tilde\sigma_1)\wedge dz_1\wedge dz_2, \\ F_4(\sigma_2) &= 3dt\wedge(\sigma_2^{-2}d\tilde\sigma_2)\wedge dz_3\wedge dz_4, \\ F_4(\sigma_3) &= 3dt\wedge(\sigma_3^{-2}d\tilde\sigma_3)\wedge dz_5\wedge dz_6, \end{aligned} \tag{42}$$

where the harmonic functions $\sigma_i, \tilde\sigma_i$ for the three 2-branes are

$$\begin{aligned} \sigma_i &= 1+\frac{2m\sinh^2\alpha_i}{R^2}, \\ \tilde\sigma_i &= 1+\frac{m\sinh 2\alpha_i}{R^2}, \quad i=1,2,3 \end{aligned} \tag{43}$$

Eqs. (42, 43) show that the expressions for the field strengths $F_4(\sigma_i)$ are different from those in [7]; here two sets of harmonic functions, σ_i and $\tilde\sigma_i$, are exploited to describe $F_4(\sigma_i)$. The dual forms in 11 dimensions of $F_4(\sigma_i)$ are given by

$$\begin{aligned} \hat{*}F_4(\sigma_1) &= 2m(\sinh 2\alpha_1)\epsilon_3\wedge dz_3\wedge dz_4\wedge dz_5\wedge dz_6, \\ \hat{*}F_4(\sigma_2) &= 2m(\sinh 2\alpha_2)\epsilon_3\wedge dz_1\wedge dz_2\wedge dz_5\wedge dz_6, \\ \hat{*}F_4(\sigma_3) &= 2m(\sinh 2\alpha_3)\epsilon_3\wedge dz_1\wedge dz_2\wedge dz_3\wedge dz_4, \end{aligned} \tag{44}$$

where ϵ_3 is the volume 3-form of S^3. The electric charges are then

$$\begin{aligned} q_e^{(1)} &= \frac{1}{\sqrt{2}\kappa}\int \hat{*}F_4(\sigma_1) = \frac{4\pi^2 m\sinh 2\alpha_1}{\sqrt{2}\kappa}S_{34}S_{56}, \\ q_e^{(2)} &= \frac{1}{\sqrt{2}\kappa}\int \hat{*}F_4(\sigma_2) = \frac{4\pi^2 m\sinh 2\alpha_2}{\sqrt{2}\kappa}S_{12}S_{56}, \\ q_e^{(3)} &= \frac{1}{\sqrt{2}\kappa}\int \hat{*}F_4(\sigma_3) = \frac{4\pi^2 m\sinh 2\alpha_3}{\sqrt{2}\kappa}S_{12}S_{34}, \end{aligned} \tag{45}$$

where the internal coordinates $z_{1,2}, z_{3,4}, z_{5,6}$ are compactified on $T^2 \times T^2 \times T^2$, and the areas of three 2-tori are S_{12}, S_{34}, S_{56} respectively. Thus the solution (41) contains seven parameters: $m, \alpha_i, i = 1, 2, 3, S_{12}, S_{34}, S_{56}$.

The ADM mass and the pressures of the solution (41) are

$$\begin{aligned} M_{ADM} &= \frac{2\pi^2 m}{\kappa_2} S_{12}S_{34}S_{56}(\cosh 2\alpha_1 + \cosh 2\alpha_2 + \cosh 2\alpha_3) \\ P_{12} &= \frac{2\pi^2 m}{\kappa_2} S_{12}S_{34}S_{56}(-2\cosh 2\alpha_1 + \cosh 2\alpha_2 + \cosh 2\alpha_3) \\ P_{34} &= \frac{2\pi^2 m}{\kappa_2} S_{12}S_{34}S_{56}(\cosh 2\alpha_1 - 2\cosh 2\alpha_2 + \cosh 2\alpha_3) \\ P_{56} &= \frac{2\pi^2 m}{\kappa_2} S_{12}S_{34}S_{56}(\cosh 2\alpha_1 + \cosh 2\alpha_2 - 2\cosh 2\alpha_3) \end{aligned} \tag{46}$$

As discussed in former sections, the pressures are related to each other by the constraint

$$P_{12} + P_{34} + P_{56} = 0 \tag{47}$$

which shows the exchange symmetry of the solution (41) with respect to the three 2-branes. Due to the presence of the constraint (47), the number of unconstrained pressures is 2. Then two unconstrained pressures, the ADM mass and the three electric charges match the six "numbers" of three 2-branes and 2-antibranes. Following the similar procedure of section 2.1, but only taking three extremal limits: $m \to 0, \alpha_i \to \pm\infty$ with $q_e^{(i)}$ and $\alpha_j (j \neq i)$ fixed, we can define the "numbers" of three 2-branes and anti-2-branes as follows

$$\begin{aligned} N_2^{(1)} &= \frac{\sqrt{2}\pi^2 m S_{34}S_{56}}{\kappa\hat{e}} e^{2\alpha_1}, \quad \bar{N}_2^{(1)} = \frac{\sqrt{2}\pi^2 m S_{34}S_{56}}{\kappa\hat{e}} e^{-2\alpha_1}, \\ N_2^{(2)} &= \frac{\sqrt{2}\pi^2 m S_{12}S_{56}}{\kappa\hat{e}} e^{2\alpha_2}, \quad \bar{N}_2^{(2)} = \frac{\sqrt{2}\pi^2 m S_{12}S_{56}}{\kappa\hat{e}} e^{-2\alpha_2}, \\ N_2^{(3)} &= \frac{\sqrt{2}\pi^2 m S_{12}S_{34}}{\kappa\hat{e}} e^{2\alpha_3}, \quad \bar{N}_2^{(3)} = \frac{\sqrt{2}\pi^2 m S_{12}S_{34}}{\kappa\hat{e}} e^{-2\alpha_3}. \end{aligned} \tag{48}$$

Then the three electric charges are simply

$$q_e^{(i)} = \left(N_2^{(i)} - \bar{N}_2^{(i)}\right)\hat{e}, \ i = 1, 2, 3 \tag{49}$$

and the ADM mass is

$$M_{ADM} = \frac{\hat{e}}{\sqrt{2}\kappa}\left[S_{12}\left(N_2^{(1)} + \bar{N}_2^{(1)}\right) + S_{34}\left(N_2^{(2)} + \bar{N}_2^{(2)}\right) + S_{56}\left(N_2^{(3)} + \bar{N}_2^{(3)}\right)\right] \tag{50}$$

and the values of moduli are

$$S_{12} = \left(\frac{\kappa\hat{e}}{\sqrt{2}\pi^2}\right)^{\frac{1}{2}} \left(\frac{N_2^{(2)}\bar{N}_2^{(2)}N_2^{(3)}\bar{N}_2^{(3)}}{N_2^{(1)}\bar{N}_2^{(1)}}\right)^{\frac{1}{4}} m^{-\frac{1}{2}},$$

$$
\begin{aligned}
S_{34} &= \left(\frac{\kappa\hat{e}}{\sqrt{2}\pi^2}\right)^{\frac{1}{2}} \left(\frac{N_2^{(3)}\bar{N}_2^{(3)}N_2^{(1)}\bar{N}_2^{(1)}}{N_2^{(2)}\bar{N}_2^{(2)}}\right)^{\frac{1}{4}} m^{-\frac{1}{2}}, \\
S_{56} &= \left(\frac{\kappa\hat{e}}{\sqrt{2}\pi^2}\right)^{\frac{1}{2}} \left(\frac{N_2^{(1)}\bar{N}_2^{(1)}N_2^{(2)}\bar{N}_2^{(2)}}{N_2^{(3)}\bar{N}_2^{(3)}}\right)^{\frac{1}{4}} m^{-\frac{1}{2}}, \qquad (51)
\end{aligned}
$$

which means that the values of the moduli are functions of the nonextremality parameter m and the "numbers" of 2-branes and anti-2-branes, which is different from the situation in the D-brane picture. Eqs. (48 - 51) reveal that the original seven parameters in the solution (41) can be replaced by six "numbers" of three 2-branes and anti-2-branes plus the parameter m.

The five-dimensional Einstein-frame metric obtained by reduction of the internal coordinates $z_1, ..., z_6$ is

$$
ds_5^2 = -f^2(R)\left(1-\frac{2m}{R^2}\right)dt^2 + f^{-1}(R)\left[\left(1-\frac{2m}{R^2}\right)^{-1} dR^2 + R^2 d\Omega_3^2\right] \qquad (52)
$$

with

$$
f(R) = \frac{R^2}{\left[(R^2+2m\sinh^2\alpha_1)(R^2+2m\sinh^2\alpha_2)(R^2+2m\sinh^2\alpha_3)\right]^{\frac{1}{3}}} \qquad (53)
$$

Now the Bekenstein-Hawking entropy can be obtained from (41), (52) and (53) and is found to be

$$
S_{BH} = \frac{2\pi A_9}{\kappa^2} = \frac{2\pi A_3}{\kappa_5^2} = \frac{8\sqrt{2}\pi^3 m^{3/2}}{\kappa^2} S_{12}S_{34}S_{56} \times \cosh\alpha_1 \times \cosh\alpha_2 \times \cosh\alpha_3 \qquad (54)
$$

where $\kappa_5^2/8\pi$ is Newton's constant in $D = 5$ dimensions, and the Hawking temperature is

$$
T_H = \left(2\pi\sqrt{2m}\cosh\alpha_1\cosh\alpha_2\cosh\alpha_3\right)^{-1} \qquad (55)
$$

In terms of N's, Eq. (54) can be rewritten as

$$
S_{BH} = 2\pi\left(\sqrt{N_2^{(1)}}+\sqrt{\bar{N}_2^{(1)}}\right)\left(\sqrt{N_2^{(2)}}+\sqrt{\bar{N}_2^{(2)}}\right)\left(\sqrt{N_2^{(3)}}+\sqrt{\bar{N}_2^{(3)}}\right) \qquad (56)
$$

which shows that the black hole entropy depends only on the "numbers" of 2-branes and anti-2-branes, but is independent of the nonextremality parameter m.

3.2 The "boosted" $2 \perp 5$ configuration

The configuration of a 2-brane intersecting with a 5-brane with a "boost" along the common string is given by

$$
\begin{aligned}
ds_{11}^2 &= \sigma^{\frac{1}{3}}\omega^{\frac{2}{3}}\Big\{(\sigma\omega)^{-1}\left[-\eta^{-1}\left(1-\frac{2m}{R^2}\right)dt^2 + \eta dz_1'^2\right] \\
&\quad \sigma^{-1}dz_2^2 + \omega^{-1}(dz_3^2+dz_4^2+dz_5^2+dz_6^2) \\
&\quad + \left(1-\frac{2m}{R^2}\right)^{-1} dR^2 + R^2 d\Omega_3^2\Big\}
\end{aligned}
$$

with

$$F_4 = F_4(\sigma) + F_4(\omega) \tag{57}$$

where

$$\begin{aligned} F_4(\sigma) &= 3dt \wedge (\sigma^{-2} d\tilde{\sigma}) \wedge dz_1' \wedge dz_2, \\ F_4(\omega) &= 3(*d\tilde{\omega}) \wedge dz_2 \end{aligned} \tag{58}$$

where the harmonic functions $\sigma(\tilde{\sigma}), \omega(\tilde{\omega}), \eta$ and the differential dz_1' are defined as

$$\begin{aligned} \sigma &= 1 + \frac{2m \sinh^2 \alpha}{R^2}, \quad \tilde{\sigma} = 1 + \frac{m \sinh 2\alpha}{R^2}, \\ \omega &= 1 + \frac{2m \sinh^2 \beta}{R^2}, \quad \tilde{\omega} = 1 + \frac{m \sinh 2\beta}{R^2}, \\ \eta &= 1 + \frac{2m \sinh^2 \delta}{R^2}, \\ dz_1' &= dz_1 - \frac{m \sinh 2\delta}{R^2} \eta dt \end{aligned} \tag{59}$$

Then the electric and magnetic charges are given by

$$\begin{aligned} q_e &= \frac{1}{\sqrt{2}\kappa} \int \hat{*} F_4(\sigma) = \frac{4\pi^2 m \sinh 2\alpha}{\sqrt{2}\kappa} V_4, \\ q_m &= \frac{1}{\sqrt{2}\kappa} \int F_4(\omega) = \frac{4\pi^2 m \sinh 2\beta}{\sqrt{2}\kappa} l_2, \end{aligned}$$

with

$$Q = m \sinh 2\delta = \frac{2\kappa_5^2}{2\omega_3} \cdot \frac{2\pi(n - \bar{n})}{l_1} = \frac{\kappa^2 (n - \bar{n})}{\pi l_1^2 l_2 V_4} \tag{60}$$

where $n, \bar{n}$ are positive integers, and the internal coordinates $z_1, z_2, z_{3,4,5,6}$ are compactified on $S^1 \times S^1 \times T^4$, the circumferences of two circles and the volume of the 4-torus being taken to be l_1, l_2, V_4 respectively. The solution (57) therefore has seven parameters: $m, \alpha, \beta, \delta, l_1, l_2, V_4$.

The corresponding ADM mass and pressures of the solution (47) are found to be

$$\begin{aligned} M_{ADM} &= \frac{2\pi^2 m}{\kappa^2} l_1 l_2 V_4 (\cosh 2\alpha + \cosh 2\beta + \cosh 2\delta), \\ P_1 &= \frac{2\pi^2 m}{\kappa^2} l_1 l_2 V_4 (-2\cosh 2\alpha - \cosh 2\beta + 3\cosh 2\delta), \\ P_2 &= \frac{2\pi^2 m}{\kappa^2} l_1 l_2 V_4 (-2\cosh 2\alpha + 2\cosh 2\beta), \\ P_3 &= \frac{2\pi^2 m}{\kappa^2} l_1 l_2 V_4 (\cosh 2\alpha - \cosh 2\beta). \end{aligned} \tag{61}$$

The above three pressures are not independent, but restricted by the constraint

$$P_2 + 2P_3 = 0 \tag{62}$$

Proceeding along similar lines as in section 2.1, the "numbers" of 2-branes (anti-2-branes), 5-branes (anti-5-branes) and right (left)-moving strings can be defined as

$$\begin{aligned}
N_2 &= \frac{\sqrt{2}\pi^2 m V_4}{\kappa\hat{e}} e^{2\alpha}, \quad \bar{N}_2 = \frac{\sqrt{2}\pi^2 m V_4}{\kappa\hat{e}} e^{-2\alpha}, \\
N_5 &= \frac{\sqrt{2}\pi^2 m l_2}{\kappa\hat{e}_m} e^{2\beta}, \quad \bar{N}_5 = \frac{\sqrt{2}\pi^2 m l_2}{\kappa\hat{e}_m} e^{-2\beta}, \\
n &= \frac{\pi m l_1^2 l_2 V_4}{2\kappa^2} e^{2\delta}, \quad \bar{n} = \frac{\pi m l_1^2 l_2 V_4}{2\kappa^2} e^{-2\delta}
\end{aligned} \tag{63}$$

In terms of these the electric and magnetic charges are

$$q_e = (N_2 - \bar{N}_2)\hat{e}, \; q_m = (N_5 - \bar{N}_5)\hat{e}_m, \; q = (n - \bar{n})\hat{e}. \tag{64}$$

Also the ADM mass is

$$M_{ADM} = \frac{1}{\sqrt{2}\kappa}\left[l_1 l_2 \hat{e}(N_2 + \bar{N}_2) + l_1 V_4 \hat{e}_m (N_5 + \bar{N}_5) + \frac{2\sqrt{2}\kappa\pi}{l_1}(n + \bar{n})\right] \tag{65}$$

and the values of moduli are

$$\begin{aligned}
l_1 &= \sqrt{2}\pi\left(\frac{n\bar{n}}{N_2\bar{N}_2 N_5\bar{N}_5}\right)^{\frac{1}{4}} m^{\frac{1}{2}}, \\
l_2 &= \frac{\kappa\hat{e}_m}{\sqrt{2}\pi^2}\sqrt{N_5\bar{N}_5}\, m^{-1}, \\
V_4 &= \frac{\kappa\hat{e}}{\sqrt{2}\pi^2}\sqrt{N_2\bar{N}_2}\, m^{-1},
\end{aligned} \tag{66}$$

which displays that there are constraints among the moduli due to the dependence of the mass of the 2-brane on that of 5-brane. Eqs. (63-66) imply that the original seven parameters in (57) can be replaced by six "numbers" of 2-branes (anti-2-branes), 5-branes (anti-5-branes), and right (left)-moving strings, plus the parameter m.

Compactifying the brane-coordinates $z_i, i = 1, ..., 6$, one obtains the five-dimensional Einstein-frame metric which has the form of (52), but with

$$f(R) = \frac{R^2}{\left[(R^2 + 2m\sinh^2\alpha)(R^2 + 2m\sinh^2\beta)(R^2 + 2m\sinh^2\delta)\right]^{\frac{1}{3}}} \tag{67}$$

Then the Bekenstein-Hawking entropy is

$$S_{BH} = \frac{2\pi A_9}{\kappa^2} = \frac{2\pi A_3}{\kappa_5^2} = \frac{8\sqrt{2}\pi^3 m^{3/2}}{\kappa^2} l_1 l_2 V_4 \cdot \cosh\alpha \cdot \cosh\beta \cdot \cosh\delta, \tag{68}$$

and the Hawking temperature is

$$T_H = (2\pi\sqrt{2m}\cosh\alpha\cosh\beta\cosh\delta)^{-1} \tag{69}$$

Reexpressing (68) in terms of the "numbers" of M-branes and anti-M-branes, we have

$$S_{BH} = 2\pi \left(\sqrt{N_2} + \sqrt{\bar{N}_2}\right) \left(\sqrt{N_5} + \sqrt{\bar{N}_5}\right) \left(\sqrt{n} + \sqrt{\bar{n}}\right) \tag{70}$$

which means that the black hole entropy depends only on the "numbers" of the 2-(anti)-branes, 5-(anti)-branes, and the right (left)-moving strings, but is independent of the nonextremality parameter m. From (26), (40), (56) and (70), we find that in terms of the "numbers" of M-branes and anti-M-branes, the prefactors of the Bekenstein-Hawking entropies for nonextremal $4D$ and $5D$ black holes are model independent [28].

4 Microscopic explanation

In M-theory, 5-branes interact via exchange of membranes with boundaries on the 5-branes [29]-[31], and quantization of the boundary states results in the 5-brane low-energy effective theory [18, 32], which suggests that collapsed membranes live on the intersection manifold of our intersecting 5-branes and bind them together [22]. As an example, we consider the "boosted" $5 \perp 5 \perp 5$ M-brane solution, and associate the microscopic massless states with those of 2-branes attached to 5-branes near the intersection point. In particular, one can visualize a 2-brane with three holes, each of them attached to different 5-dimensional hyperplanes in which the 5-branes lie, and for the three 5-branes intersecting along a string, the collapsed membrane gives the desired string, the momentum of the membranes then becoming the momentum of the string. The generating function of the degeneracy of states with momentum $2\pi n/l_1$ is then given by [16, 13]

$$\sum_n d(n) q^n = \frac{\pi_n (1+q^n)^K}{\pi_n (1-q^n)^K} \tag{71}$$

with

$$K = 4 N_5^{(1)} N_5^{(2)} N_5^{(3)} \tag{72}$$

which means that the collapsed membranes act like a single self-dual string with 4 bosonic and 4 fermionic zero modes. The degeneracy of states is then $d(n) = \exp\left[2\pi\sqrt{n N_5^{(1)} N_5^{(2)} N_5^{(3)}}\right]$ for large n, and the entropy

$$S_{stat} = \ln d(n) = 2\pi \sqrt{n N_5^{(1)} N_5^{(2)} N_5^{(3)}} \tag{73}$$

Considering now the near-extremal black hole, when $\bar{N}_5^{(1)} = \bar{N}_5^{(2)} = \bar{N}_5^{(3)} = 0$ and l_1 is large, the lightest excitations will be the momentum modes. At weak coupling, the entropy has the form

$$S_{stat} = 2\pi \sqrt{N_5^{(1)} N_5^{(2)} N_5^{(3)}} \left(\sqrt{n} + \sqrt{\bar{n}}\right) \tag{74}$$

where $\bar{n}$ denotes the momentum of the left movers. Since U-duality interchanges the different branes and strings, a result similar to (74) with the indices permuted is obtained for these cases. Thus the entropy (40) is the simplest duality invariant expression which is compatible with the different nonextremal limits.

The above algorithm holds for the nonextremal $5D$ black hole obtained upon toroidal compactification from the "boosted" $2 \perp 5$ nonextremal intersecting M-brane solution. From the symmetry transformation, which is a combination of T-duality and SL(2,Z) symmetry of the $D = 10$ type IIB string theory lifted to $D = 11$ M-theory, we can obtain the statistical entropy for the other two nonextremal black holes obtained from $2 \perp 2 \perp 5 \perp 5$ and $2 \perp 2 \perp 2$ nonextremal intersecting M-brane configurations.

5 Conclusion

In the above, the D-brane and anti-D-brane picture has been generalized to an M-brane and anti-M-brane one, and the nonextremal $4D$ and $5D$ black holes obtained upon toroidal compactification from the nonextremal intersecting M-brane solutions have been identified as a collection of M-branes and anti-M-branes. In our improved nonextremal intersecting M-brane solutions, two sets of harmonic functions were first introduced, which makes it easy to explicitly calculate the electric charges. In a unified frame of $D = 11$ M-theory, the "numbers" of M-branes and anti-M-branes have been defined consistently. The constraints among the pressures, here found for the first time, reflect the exchange symmetry of the nonextremal intersecting M-brane solutions, and only the unconstrained pressures, ADM mass and gauge charges match the "numbers" of M-branes and anti-M-branes. Unlike the D-brane and anti-D-brane picture in which the moduli depend only on the "numbers" of D-branes and anti-D-branes, the new features in the M-brane and anti-M-brane picture have been shown to be: 1) there are constraints among the moduli, and some moduli can be chosen as free parameters; 2) the moduli are functions of the nonextremality parameter m and the "numbers" of M-branes and anti-M-branes. As a result, these new features resolve the discrepancy in the number of free parameters in the D-brane picture. In terms of the "numbers" of M-branes and anti-M-branes, the prefactors of the entropies have been found to be model independent, and the Bekenstein-Hawking entropy assumes the $E_7(E_6)$ invariant form for $4D$ ($5D$) black holes. The microscopic origin of the Bekenstein-Hawking entropy for the nonextremal black holes has been explained from the M-brane and anti-M-brane picture. All of these features together seem to indicate that the extension of the D-brane and anti-D-brane picture to the M-brane and anti-M-brane one is quite convincing. However, such an M-brane and anti-M-brane picture is invalid for the analysis of the nonextremal black holes in $6 \leq D \leq 9$ dimensions, where the "numbers" of M-branes and anti-M-branes cannot be defined consistently, and the black hole entropy depends on the nonextremality parameter m.

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References

[1] P.K. Townsend, Phys. Lett **B350** (1995) 184; C.M. Hull and P.K. Townsend, Nucl. Phys. **B438** (1995) 109; E. Witten, Nucl. Phys. **B443** (1995) 85; J.H. Schwarz, Phys. Lett. **B367** (1996) 97; K. Becker, M. Beker and A. Strominger, Nucl. Phys. **B456** (1995) 130; P.Hořava and E. Witten, Nucl. Phys. **B460** (1996) 506; I. Bars and S. Yankielowicz, hep-th/9511098.

[2] E. Cremmer, B. Julia and J. Scherk, Phys. Lett. **B76** (1978) 409.

[3] M.J. Duff and K.S. Stelle, Phys. Lett. **B253** (1991) 113.

[4] R. Güven, Phys. Lett. **B276** (1992) 49.

[5] G. Papadopoulos and P.K. Townsend, hep-th/9603087.

[6] A.A. Tseytlin, hep-th/9604035.

[7] M. Cvetič and A.A. Tseytlin, hep-th/9606033.

[8] G.T. Horowitz, J. Maldacena and A. Strominger, hep-th/9603109.

[9] G.T. Horowitz, D.A: Lowe and J.M. Maldacena, hep-th/9603195.

[10] R. Kallosh and A. Rajaraman, hep-th/9604193.

[11] A. Strominger and C. Vafa, hep-th/9601029.

[12] G.T. Horowitz and A. Strominger, hep-th/9602051; A. Strominger hep-th/9602111.

[13] C. Callan and J. Maldacena, hep-th/9602043.

[14] A. Stominger and J. Maldacena, hep-th/9603060; C.V. Johnson, R.R. Khuri and R.C. Myers, hep-th/9603061; J.C. Breckenridge, R.C. Myers, A.W. Peet and C. Vafa, hep-th/9602065.

[15] J. Maldacena, hep-th/9607235.

[16] V. Balasubramian and F. Larsen, hep-th/9604189; M.B. Green and M. Gutperle, hep-th/9604091.

[17] K. Behrndt, E. Bergshoeff, B. Janssen, hep-th/9604168; K. Behrndt and E. Bergshoeff, hep-th/9605216; E. Bergshoeff, hep-th/9607238.

[18] R. Dijkgraaf, E. Verlinde and H. Verlinde, hep-th/9603126; hep-th/9604055; hep-th/9607026.

[19] J. Polchinski, S. Chundhuri and C.V. Johnson, hep-th/9602052.

[20] R. Gregory and R. Laflamme, Phys. Rev. Lett. **70** (1993) 2837; Nucl. Phys. **B428** (1994) 399.

[21] M. Cvetič and C.M. Hull, hep-th/9606193.

[22] I.R. Klebanov and A.A. Tseytlin, hep-th/9604166; hep-th/9607107.

[23] J.X. Lu, Phys. Lett. **B313** (1993) 29.

[24] S.P. de Alwis, hep-th/9607011.

[25] A.A. Tseytlin, hep-th/9601177; M. Cvetič and A. Tseytlin, hep-th/9512031; F. Larsen and F. Wilczek, hep-th/9511064.

[26] S. Ferrara, R. Kallosh and A. Strominger, Phys. Rev. **D52** (1995) 5412; M. Cvetič and D. Youm, hep-th/9507090; G. Gibbons and P. Townsend, Phys. Rev. Lett. **71** (1993) 3754.

[27] A. Dabholkar, J.P. Gauntlett, J.A. Harvey and D. Waldram, hep-th/9511053.

[28] J.–G. Zhou, F. Zimmerschied, J.–Q. Liang and H.J.W. Müller- Kirsten, Phys. Lett. **B359** (1995) 62.

[29] K. Becker and M. Becker, hep-th/9602071.

[30] A. Strominger, hep-th/ 9512059.

[31] P.K. Townsend, hep-th/9512062.

[32] E.J. Martinec, hep-th/9608017.